

4/20/17 "I am not a product of my circumstances. I am a product of my decisions."-Stephen Covey

HW: "Geometric Sequences and Series" Finish the packet

AIM: What is a Geometric Sequence?

Warm Up:

Given the sequence $3, 9, 27, 81, \dots$
(a) Find the tenth term.

59049

(b) Can you write a formula to help you find any given term of the sequence?

$$a_n = 3(3)^{n-1}$$

Starting Value

number we multiply by over and over (common ratio)

A sequence is **geometric** if the ratio of consecutive terms are the same (multiply by a nonzero constant). A geometric sequence comes in the form $a_1, a_1r, a_1r^2, a_1r^3, \dots$ where a_1 is the first term and r is the common ratio. The n th term of a geometric sequence is

$a_n = a_1r^{n-1}$. If the geometric sequence is finite, the sum of the first n terms or the n th

partial sum is $S_n = \frac{a_1(1-r^n)}{1-r}$

specific term formula

sum formula

a_1 = 1st term

r = common ratio

n = number of terms

1. Determine if the following sequences are geometric.

(a) 5, 10, 20, 40, ...

yes

b/c multiply by 2 each time.

(b) 1, 2, 6, 24, ...

No b/c we are not multiplying by the same number every time.

2.. Write a rule for the n th term of the geometric sequence $6, -3, \frac{3}{2}, -\frac{3}{4}, \dots$ and find a_6 .

$$a_n = a_1(r)^{n-1}$$

$$a_1 = 6$$

$$r = -\frac{1}{2}$$

$$\frac{-3}{6} = -\frac{1}{2} = r$$

⊗ Find "r" by dividing any term by the one to its left.

Formula:

$$a_n = 6\left(-\frac{1}{2}\right)^{n-1}$$

$$a_6 = 6\left(-\frac{1}{2}\right)^{6-1}$$

$$a_6 = -\frac{3}{16}$$

3. Find the 7th term of the sequence $2, 6, 18, 54, \dots$

$$a_7 = 2(3)^{7-1} \quad r=3$$

$$= 2(3)^6$$

$$= \boxed{1458}$$

4. Given that $a_1 = 5$ and $a_2 = 15$ are the first two terms of a geometric sequence, determine the values of a_3 and a_{10} . Show the calculations that lead to your answers.

$$\underline{5} \quad \underline{15 \times 3} \quad \underline{45} \quad \dots \quad \underline{98415}$$

$$r = \frac{15}{5} = 3$$

$$a_n = 5(3)^{n-1}$$

$$a_{10} = 5(3)^{10-1}$$

$$= 98415$$

5. Find the sum of the first $\hat{8}$ terms of the sequence geometric $-5, 15, -45, 135, \dots$

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad r = -3$$

$$r = (-3)$$

$$n = 8$$

$$a_1 = (-5)$$

$$Sum = \frac{(-5)(1-(-3)^8)}{1-(-3)} = \boxed{8200}$$

6. Write the first five terms of the geometric sequence whose first term is $\frac{1}{5}$ and whose common ratio is $-\frac{1}{5}$.

$$\underline{\frac{1}{5}} \quad \underline{-\frac{1}{25}} \quad \underline{\frac{1}{125}} \quad \underline{-\frac{1}{625}} \quad \underline{\frac{1}{3125}}$$

$$\frac{1}{5} \left(-\frac{1}{5}\right) = -\frac{1}{25} \left(-\frac{1}{5}\right) = \frac{1}{125} \left(-\frac{1}{5}\right) = -\frac{1}{625} \left(-\frac{1}{5}\right) = \frac{1}{3125}$$

7. Find the n^{th} term of the geometric sequence whose 1st term is 20 and whose common ratio is 2.

$$\begin{aligned} n &= 15 \\ a_1 &= 20 \\ r &= 2 \\ a_{15} &= 20(2)^{15-1} \\ &= 327,680 \end{aligned}$$

8. The first term of a geometric sequence is 8, and the second term is 4. Find the fifth term.

9. The 4th term of a geometric sequence is 125 and the 10th term is $\frac{125}{64}$. Find the 14th term.

$10 - 4 = 6$

--- 125 --- $\frac{125}{64}$ --- 4 times

6 spots

$$\frac{125}{64} = \frac{125}{125} r^6$$

$$\frac{1}{64} = r^6$$

$$\sqrt[6]{\frac{1}{64}} = \sqrt[6]{r^6}$$

$$\frac{1}{2} = r$$

$$a_n = 125\left(\frac{1}{2}\right)^{10}$$

$$a_n = \frac{125}{1024}$$

$$a_n = \frac{125}{64}\left(\frac{1}{2}\right)^4 = \frac{125}{1024}$$

10. Find the 7th term of the geometric sequence whose third term is $\frac{16}{3}$ and whose fifth term is $\frac{64}{27}$.

11. Which term of the geometric sequence 2, 6, 18, ... is 118,098?

12. The second and fifth terms of a geometric sequence are 10 and 1250 respectively. Is 31,250 a term of this sequence? If so, which term is it?

13. Find the sum of each:

(a) 1, 3, 9, ..., 2187

geometric

Find n :

$$2187 = 1(3)^{n-1}$$

$$2187 = 3^{n-1}$$

$$\log_3 2187 = n-1$$

$$(\log_3 2187) + 1 = n$$

$$n = 8$$

$a_1 = 1$

$r = 3$

$n = 8$

$\text{Sum} = \frac{1(1-3^8)}{1-3}$

$\text{Sum} = 3280$

(b) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \dots, -\frac{1}{512}$

14. In a geometric sequence, it is known that $a_1 = -1$ and $a_4 = 64$. The value of a_{10} is

- (1) -65,536 (3) 512
(2) 262,144 (4) -4096

$$\boxed{125 = a_1 \left(\frac{1}{2}\right)^3}$$
$$8 \cdot 125 = a_1 \left(\frac{1}{8}\right)^8$$
$$1000 = a_1$$

15. Generate the next **three** terms of each geometric sequence defined below.

(a) $a_1 = -8$ with $r = -1$

(b) $a_n = a_{n-1} \cdot \frac{3}{2}$ and $a_1 = 16$

(c) $f(n) = f(n-1) \cdot -2$ and $f(1) = 5$

16. Generate the next three terms of the geometric sequences given below.

(a) $a_1 = 4$ and $r = 2$

(b) $f(n) = f(n-1) \cdot \frac{1}{3}$ with $f(1) = 9$

(c) $t_n = t_{n-1} \cdot \sqrt{2}$ with $t_1 = 3\sqrt{2}$